S.-T. Yau College Student Mathematics Contests 2021

Oral Exams in Geometry and Topology

All-round (2 problems)

1. Let Σ be a regular surface in $\mathbb{R}^3$, and ν be its Gauss map. Let $f : \Sigma \to (0, \infty)$ be a smooth function on Σ, and consider the set:

$$\hat{\Sigma} = \{p + f(p)\nu(p) : p \in \Sigma\}.$$

Show that if $\mathrm{id} + fh : T_p\Sigma \to T_p\Sigma$ is invertible for all $p \in \Sigma$, then $\hat{\Sigma}$ is also a regular surface. Here h is the second fundamental form.

2. Suppose M is a compact manifold with boundary. Glue together a pair of M's along their boundaries to form a new manifold $\widetilde{M}$. Explain how to use a Mayer-Vietoris sequence to find a relation between $\chi(\widetilde{M})$, $\chi(\partial M)$ and $\chi(M)$.